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During the period April. 15 1989, AFOSR, supported research work on the stochastic behavior of the LMS and related adaptive algorithms has yielded results in two major areas: (A) Digital Implementation of Stochastic Gradient Type adaptive Algorithms; (B) LMS and RLS Performance Comparison for Tracking a Chirped Sinusoid in Noise.

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During the period April. 15, 1988 - May 15, 1989, AFOSR supported research work on the the stochastic behavior of the LMS and related adaptive algorithms has yielded results in two major areas:

A. Digital Implementation of Stochastic Gradient Type Adaptive Algorithms

The mathematical models developed in [15] were extended to analyze the behavior of four types of digital implementations of the basic LMS algorithm.

a) Nonlinear quantization effects in LMS and Block LMS adaptation were compared on the basis of dynamic range, algorithm transient response and stalling phenomena. It was shown [3] that the LMS algorithm requires $(1/2 \log_2 L - K)$ fewer bits than the BLMS algorithm for the same saturation and stalling effects (L =block length and K lies between .2 and 1).

b) Saturation effects in the LMS Adaptive Line Enhancer (ALE) were studied using the saturation model developed in [15]. The ability of a digitally implemented ALE to cancel a weak sinusoid in the presence of a strong sinusoid and noise was investigated [4]. The mathematical model and the simulations both showed a significant slowdown in cancellation of the weaker sinusoid when the larger sinusoid caused the algorithm to operate in saturation.

c) [15] studied the effect of a saturation non-linearity on the error term in the weight up-date term in LMS adaptation. [15] was extended to consider the effect of a saturation non-linearity on the entire weight up-date term [5]. By comparison with the [15], there is no significant difference in the behavior of digital implementations of the LMS algorithm whether round-off occurs before or after multiplying the error by the data.

d) [15] was also extended to the study of LMS Echo-Cancellation when the data is non-gaussian [6]. It had been previously shown that, when the LMS algorithm is implemented using a sign detector (one bit quantizer) for the error, serious performance degradation occurs for binary data as compared to gaussian data. The saturation nonlinearity of [15] was used to study the performance of the binary and gaussian data models. The number of additional bits for binary data for the same performance was evaluated.

B. LMS and RLS Performance Comparison for Tracking a Chirped Sinusoid in Noise

The RLS adaptation algorithm is well-known to have a faster transient response than the LMS algorithm when the input data is colored and stationary. It is not clear that RLS is superior to LMS for tracking non-stationary inputs. The chirped sinusoid is an example of a non-stationary input for which it is useful to obtain a performance comparison of the two algorithms.

[8] studies the ability of the LMS adaptive algorithm to track a fixed amplitude complex chirped exponential buried in additive white gaussian noise. The exponential is recovered using an M-tap predictor W (adaptive line enhancer). When W is controlled by the LMS algorithm with forgetting rate $\nu = \mu P_n$ (P_n is the input noise power), the output misadjustment is dominated by a lag term of order ν^{-2} and a fluctuation term of order ν . A value ν_{opt} exists which yields a minimum misadjustment \mathcal{M}_{min} . ν_{opt} and \mathcal{M}_{min} were evaluated as a function of the signal chirp rate ψ , the number of taps M , the noise power P_n and the signal-to noise ratio ρ .

[7] studies the ability of the exponentially weighted RLS adaptive algorithm to track a fixed amplitude complex chirped exponential buried in additive white gaussian noise. The exponential is adaptively recovered using an M-tap predictor W . Five principal results of this work are: 1) the performance of the algorithm, 2) the methodology of the analysis, 3) proof of the quasi-deterministic nature of the data-covariance estimates, 4) new analysis of RLS for an inverse system modelling problem, and 5) new analysis of RLS for a deterministic time-varying model for the optimum filter. It is shown that, when W is controlled by the RLS algorithm with forgetting rate β , the output misadjustment is dominated by a delay term of order β^{-2} and a fluctuation term of order β . Thus, a value β_{opt} exists which yields a minimum misadjustment \mathcal{M}_{min} . β_{opt} and \mathcal{M}_{min} are evaluated as a function of the signal chirp rate ψ , the number of taps M , the noise power P_n and the signal-to noise ratio ρ .

The results in [7] and [8] were combined in [9] to yield tracking performance comparisons between the two algorithms. The minimum misadjustments for the two algorithms were compared for the same set of input signal parameters. For a satisfactory predictor output signal-to-noise ratio, LMS will track better than RLS (smaller misadjustment) unless $\rho \gg 1$ or ψ is sufficiently. Thus, this work leads to the very useful and unexpected result that LMS usually tracks better.

POTENTIAL AIR FORCE APPLICATIONS

A. The LMS algorithm is an extremely popular form of adaptation because of its simplicity of implementation and well-understood behavior. One would assume that it has found use in many Air Force systems where a priori statistical information about the input signals is unavailable. These applications could include echo-cancellation for hard-wire communication systems, adaptive interference cancellation for spread-spectrum communication systems, adaptive beam-forming, and adaptive noise cancellation in jamming environments. In each case, digital implementations of the algorithm require an understanding of the number of bits required for representing the individual mathematical operations that comprise the algorithm. The results of the work described in Section A help a designer to select the correct system parameters for fast and efficient implementations of the algorithm.

B. When can one use the simpler, yet more slowly converging, LMS algorithm in place of the faster converging, but significantly more difficult to implement, RLS algorithm? This analysis suggests that LMS is superior to RLS when tracking some non-stationary signals. The impact of this result on adaptive systems used by the Air Force would be to retain the simplicity of LMS for tracking environments.

LIST OF PUBLICATIONS

A. Published Journal Articles

1. "A Weighted Normalized Frequency Domain LMS Adaptive Algorithm," co-author, IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. 36, No. 7, pp. 1002-1007, July, 1988.
2. "On the Probability Density Function of the LMS Adaptive Filter Weights," co-author, IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. 37, No.1, pp.43-56, Jan. 1989.

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3. "Nonlinear Quantization Effects in the LMS and Block LMS Adaptive Algorithms-A Comparison", IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. 37, No. 10, October 1989.
4. "Error Saturation Effects in the LMS Adaptive Line Enhancer-Transient Response", co-author, IEEE Trans. on Acoustics Speech and Signal Processing, Vol. 37, No. 11, November 1989.
5. "On Weight Update Saturation Nonlinearities in LMS Adaptation", IEEE Trans. on Acoustics Speech and Signal Processing, accepted March, 1989.

C. Papers in Review

6. "Saturation Effects in LMS Adaptive Echo-Cancellation for Binary Data", co-author, accepted, subject to minor revision, IEEE Trans. on Acoustics, Speech and Signal Processing, April 1989.
7. "RLS Performance for Recovering a Chirped Sinusoid in Noise", co-author, submitted to IEEE Trans. on Acoustics, Speech and Signal Processing, May, 1989.
8. "LMS Performance for Recovering a Chirped Sinusoid in Noise", co-author, submitted to IEEE Trans. on Acoustics, Speech and Signal Processing, May, 1989.

D. Papers in Preparation

9. "Comparison of LMS and RLS Adaptive Algorithms for Tracking a Chirped Sinusoid", co-author, to be submitted to "Signal Processing".

E. Conference Papers

10. "Superiority of LMS over RLS for Tracking a Chirped Signal", co-author, IEE Colloquium on Adaptive Filtering, London UK, March 22, 1989.
11. "Performance Comparison of LMS and RLS Adaptive Algorithms for Tracking a Chirped Sinusoidal Signal in Noise", co-author, Proc. 1989 Inter. Conf. on Acoustics, Speech and Signal Processing, Paper #13.D3.2, Glasgow, Scotland, May 23-26, 1989.
12. "Effets de Saturation sur L'Algorithme LMS en Annulation d'Echo Avec Donnes Binaires" co-author, 12'th GRETSI Symp. on Signal and Image Processing, Juan-les-Pins, France, June 12-16, 1989.

F. Invited Presentations

13. "Some Stochastic-Gradient Based Algorithms and Their Analysis" Seminar on Adaptive Processing, EE and CS Dept., University of Michigan, November 3, 1988.
14. "Some Stochastic-Gradient Based Algorithms and Their Analysis" Seminar on Adaptive Processing, EE Dept., University of Pennsylvania, April 20, 1989.

G. Additional References

15. "On Error Saturation Nonlinearities in LMS Adaptation," IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. 36, No. 4, pp. 440-452, April 1988.

ABSTRACTS OF
PUBLICATIONS

A Weighted Normalized Frequency Domain LMS Adaptive Algorithm

SHAUL FLORIAN AND NEIL J. BERSHAD, FELLOW, IEEE

Abstract—This paper presents a general filtering scheme for obtaining an input power estimate for setting the convergence parameter μ separately in each frequency bin of a frequency domain LMS adaptive filter (FDAF) algorithm. A linear filtering operation is performed on the magnitude square of the input data and incorporated directly into the algorithm as a data-dependent time-varying stochastic $\mu(n)$.

The mean performance of the weighted normalized frequency domain LMS algorithm (WNFDAF) is analyzed using i.i.d. Gaussian data, and the results are validated by the Monte Carlo simulations of the algorithm.

The simulations are also used to study the weight transient behavior. The simulations suggest that short smoothing times are sufficient for rapid weight convergence without large fluctuations in the power estimates significantly affecting transient weight behavior.

I. INTRODUCTION

FREQUENCY domain implementation of the LMS adaptive filter has advantages over time implementations. Improved convergence properties and reduced computational complexity are the two main advantages [1], [2].

In applications of the frequency domain LMS adaptive filter where the input power varies dramatically over different frequency bins, some input power measurement must be incorporated in the algorithm. A recent paper [3] described a normalized frequency domain adaptive LMS filter (NFDAF). The normalization involved an estimate of the input power in each frequency bin using a uniformly weighted moving average window. The power estimate was included directly in the frequency domain LMS algorithm. The statistical behavior of the NFDAF in [3] was investigated using Gaussian data models. Closed form expressions were derived for the transient and steady-state mean weight and mean-square error per bin.

The purpose of this paper is to extend the analysis to a weighted NFDAF (WNFDAF)—an adaptive filter which incorporates an arbitrary linear weighting on the magni-

tude square of the data for input power estimation. Thus, this extended theory is useful for the normalized frequency domain LMS filter with power estimates that have recursive descriptions and/or nonuniform weighted moving average [3]–[6]. A further purpose of this paper is to study, via Monte Carlo simulations, the transient behavior of the WNFDAF when the initial settings of the power estimate are varied.

These results are a measure of how rapidly the algorithm can respond to changing input power levels—a problem which was not investigated in [3].

II. ANALYSIS

A. Mathematical Model

Using the FDAF model in [3], if the input processes to the FFT's are wide-sense stationary over the observation time, then disjoint spectral outputs are uncorrelated.¹ Assuming the inputs are joint Gaussian random processes, the disjoint bins of the FFT output provide statistically independent outputs. Thus, each complex tap is operating on independent data. Furthermore, since the FFT operations are linear operations on the joint Gaussian input sequences, the FFT outputs are jointly complex Gaussian sequences.

The weight update equation, corresponding to a single complex tap, is given by [7]

$$\begin{aligned} W(n+1) &= W(n) + \mu e(n) x^*(n) \\ &= W(n) [1 - \mu |x(n)|^2] + \mu d(n) x^*(n) \end{aligned} \quad (1)$$

where

- $W(n)$ = complex scalar weight on the n th iteration
- $e(n)$ = error waveform = $d(n) - y(n)$
- $y(n)$ = filter output = $W(n) x(n)$
- $d(n)$ = reference waveform
- $x(n)$ = input data sequence
- μ = adaptation coefficient.

The adaptive filter scheme incorporates the same input signal model as in [3]. The input sequences $d(n)$ and $x(n)$ contain a desired component, buried in statistically inde-

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S. Florian was on leave at the Department of Electrical Engineering, University of California, Irvine, CA 92717. He is with RAFAEL, Haifa, Israel.

N. J. Bershad is with the Department of Electrical Engineering, University of California, Irvine, CA 92717, on leave at the Laboratoire des Signaux et Systemes, Ecole Supérieure d'Electricité, 91190 Gif sur Yvette, France.

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¹Although this is true for white noise inputs, it is not generally true since the FFT bins are not spectrally disjoint. However, the bins are approximately uncorrelated if the power spectrum of the signal changes slowly over the bandwidth of a single FFT bin [8].

On the Probability Density Function of the LMS Adaptive Filter Weights

NEIL J. BERSHAD, FELLOW, IEEE, AND LIAN ZUO QU

Abstract—In this paper, the joint probability density function of the weight vector in LMS adaptation is studied for Gaussian data models. An exact expression is derived for the characteristic function of the weight vector at time $n + 1$ conditioned on the weight vector at time n . The conditional characteristic function is expanded in a Taylor series and averaged over the unknown weight density to yield a first-order partial differential-difference equation in the unconditioned characteristic function of the weight vector.

The equation is approximately solved for small values of the adaptation parameter and the weights are shown to be jointly Gaussian with time varying mean vector and covariance matrix given as the solution to well-known difference equations for the weight vector mean and covariance matrix. The theoretical results are applied to analyzing the use of the weights in detection and time delay estimation. Simulations, which support the theoretical results, are also presented.

1. INTRODUCTION

THE time domain LMS adaptive filter algorithm [1] has found many applications in situations where the statistics of the input processes are unknown or changing. These include noise cancelling [2], line enhancing [3], [4], [7], and adaptive array processing [5], [6]. The algorithm uses a transversal filter structure driven by a primary input (Fig. 1). The filter weights are updated iteratively based upon the difference between the filter output and a reference input, so as to minimize the mean-square error of the difference.

The LMS algorithm has been very thoroughly investigated over a long period of time. Its transient weight mean and covariance matrix and mean square error behavior have been evaluated precisely [6]–[9] for uncorrelated input data. Expressions for the transient mean square error as a function of the eigenvalues of the data covariance matrix R_{xx} have been given by a number of authors for both the real [6], [8], [9] and complex [6], [7] LMS algorithms.

Although the results in [6]–[9] are useful for determining the transient and steady-state mean and covariance of the LMS algorithm, there are many situations where additional statistical information about the weight vector would be useful. These cases include detection of a nar-

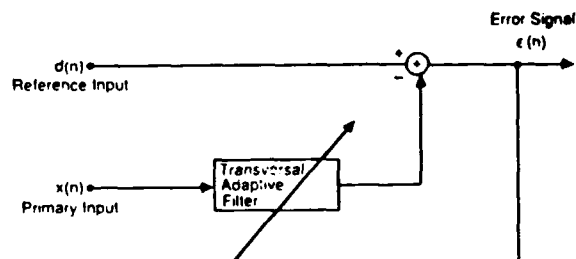


Fig. 1. LMS adaptive filter structure.

row-band line component in background noise using the weight vector as a test statistic (Adaptive Line Enhancer [7], [10]–[14]), by using the filtered output as a test statistic [13], [14], and for time delay estimation [15].

More recently, the LMS algorithm has been used as a canceller as part of a spread spectrum communication system. The output of the canceller acts as the input to a matched filter binary decision device. Knowledge of the statistics of the canceller output is crucial to predicting error probabilities for the system [16], [17], [28].

In all four cases, knowledge of the first and second moments of the weights is not sufficient to calculate 1) receiver operating characteristics (ROC's) relating detection and false alarm probabilities for the detectors, 2) error probabilities for the binary decisions, and 3) estimation performance. Most often, via a central limit argument, it is assumed that the test statistic and/or the weights are Gaussian [14], [16], [17]. In some unpublished work [18], simulations suggest that the weights are Gaussian. However, there is no existing theory supporting these simulations. Some recent theoretical work [19] on the frequency domain LMS algorithm has shown that the single complex weight is Gaussian in steady state. Further recent work has shown this result also holds in the transient case after an initial phase of adaptation [20].

In this paper, the joint characteristic function of the weight vector in real LMS adaptation is investigated when the inputs are zero mean stationary Gaussian sequences, independent from iteration to iteration. Simulations which support the theoretical analysis are also presented. The results of the analysis are applied to 1) detecting correlation between the primary and reference inputs using the weight vector as the input to a matched filter, and 2) estimating the relative delay between the primary and the reference.

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**"NONLINEAR QUANTIZATION EFFECTS IN THE LMS AND BLOCK LMS
ADAPTIVE ALGORITHMS - A COMPARISON"¹**

by

N.J. Bershad²

February 1988

Revised October 1988

1. This work was supported by the Air Force Office of Scientific Research under Project 2304/A6, Grant 86-0093.
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3

Abstract

Analog implementations of the LMS and Block LMS (BLMS) adaptive filtering algorithms have been shown to be equivalent with respect to adaptation speed and steady-state misadjustment errors. However, the BLMS algorithm offers significant reductions in computational complexity due to block processing.

In this paper, digital implementations of the two algorithms are compared with respect to finite word effects. The algorithm stalling phenomena is studied using gaussian data models and conditional expectation arguments. It is shown that the BLMS algorithm requires $(\frac{1}{2} \log_2 L - K)$ fewer bits for the same stalling behavior (L = block length and K lies between .2 and 1, depending on the precise definition of algorithm stalling). On the other hand, the LMS algorithm requires $\log_2 L$ fewer bits than BLMS for the same level of saturation behavior (transient response) at algorithm initialization. Hence, overall the LMS algorithm requires $(\frac{1}{2} \log_2 L + K)$ fewer bits than the BLMS algorithm for the same saturation and stalling effects.

"ERROR SATURATION EFFECTS IN THE LMS ADAPTIVE LINE ENHANCER-TRANSIENT RESPONSE"¹

by

S. Florian² and N.J. Bershad³

March 1988

Revised November 1988

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ABSTRACT

Digital implementation of the LMS Adaptive Line Enhancer (ALE) introduces certain nonlinear effects. This paper investigates feedback error signal saturation effects on the ALE adaptation. A set of non-linear coupled difference equations is derived by projecting the mean weight vector upon a set of orthogonal basis functions.. These equations are used to study the transient behavior of ALE for the case of one and two sinusoids in broadband noise. Simulations are presented which support the results of the theoretical model.

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(5)

**ON WEIGHT UPDATE SATURATION NONLINEARITIES
IN LMS ADAPTATION**

by

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Revised January 1989

1. This work was supported by the Air Force Office of Scientific Research under Project 2304/A6, Grant 86-0093.
2. Department of Electrical Engineering, University of California, Irvine, CA 92717.

ABSTRACT

The effect of a saturation type non-linearity in the weight update equation in LMS adaptation is investigated for a white gaussian data model. Non-linear difference equations are derived for the weight first and second moments that include the effect of a $1-e^{-x}$ saturation type non-linearity on the update term driving the algorithm. A non-linear difference equation for the mean norm is explicitly solved via a differential equation approximation and integration by quadratures. The steady-state second moment weight behavior is evaluated approximately for the nonlinearity. Using these results, the tradeoff between the extent of weight up-date saturation, steady-state excess mean-square-error and rate of algorithm convergence is studied. For the same steady-state misadjustment error, the trade-off shows that 1) starting with a sign detector, the convergence rate is increased by nearly a factor of two for each additional bit, 2) as the number of bits is increased further, the additional bits buy very little in convergence speed, asymptotically approaching the behavior of the linear model. Thus, by comparison with previous results [3], there is no significant difference in the behavior of digital implementations of the LMS algorithm whether round-off occurs before or after multiplying the error by the data.

**"SATURATION EFFECTS IN LMS ADAPTIVE
ECHO-CANCELLATION FOR
BINARY DATA"¹**

**N.J. BERSHAD^{2,3}
M. BONNET^{3,4}**

September 1988

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Abstract

The effect of a saturation type error non-linearity in the weight update equation in LMS adaptive echo-cancellation is investigated for an independent binary data model. A nonlinear difference equation is derived for the mean norm of the difference between the estimate and the unknown filter to be estimated by the algorithm. The difference equation is evaluated numerically. It is shown that far-end binary data interference is much more deleterious to algorithm transient behavior than far-end gaussian data interference. The number of additional bits for the same cancellation convergence rates for binary vs. gaussian interference of the same power is studied as a function of various system parameters.

Algorithm convergence rates are studied as a function of an arbitrary probability density function for the far-end data. It is shown that a binary pdf causes the worst degradation and a gaussian shaped pdf causes the least degradation.

**"RLS PERFORMANCE FOR RECOVERING
A CHIRPED SINUSOID IN NOISE"¹**

by

O. Macchi² and N. J. Bershad^{2,3}

May 1989

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ABSTRACT

This paper studies the ability of the exponentially weighted RLS adaptive algorithm to track a complex chirped exponential signal buried in additive white gaussian noise. The signal is adaptively recovered using an M-tap predictor W.

There are five principal results of this paper: 1) the performance of the algorithm, 2) the methodology of the analysis, 3) proof of the quasi-deterministic nature of the data-covariance estimates, 4) new analysis of RLS for an inverse system modelling problem, and 5) new analysis of RLS for a deterministic time-varying model for the optimum filter. Specifically, it is shown that,

- 1) when W is controlled by the RLS algorithm with forgetting rate $\beta = (1-\lambda)$, the output misadjustment is dominated by a delay term of order β^{-2} and a fluctuation term of order β . Thus, a value β_{opt} exists which yields a minimum misadjustment \mathcal{M}^{min} . β_{opt} and \mathcal{M}^{min} are evaluated as a function of the signal chirp rate ψ , the number of taps M, the noise power P_n and the signal-to noise ratio ρ . For sufficiently small ψ ,

$$\beta_{opt} = \{(M+1)\rho\psi^2\}^{1/3}, \quad \mathcal{M}^{min} = (3/4) P_n (M+1) \beta_{opt}.$$

- 2) The estimate of data covariance matrix R(k) satisfies

$$\lim_{k \rightarrow \infty} E\{[R(k) - E[R(k)]]^2\} \ll \{E[R(k)]\}^2$$

**"LMS Performance for
Recovering a Chirped Sinusoid in Noise" ¹**

by

N. J. Bershad^{2,3} and O. Macchi³

April 1989

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1. This work was supported by the U. S. Air Force Office of Scientific Research under Project 2304/A6, Grant 86-0093, by the National Center for Scientific Research (CNRS), Government of France and the University of Paris, Orsay.
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ABSTRACT

This paper studies the ability of the LMS adaptive algorithm to track a fixed amplitude complex chirped exponential buried in additive white gaussian noise. The exponential is recovered using an M-tap predictor W. When W is controlled by the LMS algorithm with forgetting rate $\nu = \mu P_n$ (P_n is the input noise power), the output misadjustment is dominated by a lag term of order ν^{-2} and a fluctuation term of order ν . Thus, a value ν_{opt} exists which yields a minimum misadjustment \mathcal{M}^{min} . ν_{opt} and \mathcal{M}^{min} are evaluated as a function of the signal chirp rate ψ , the number of taps M, the noise power P_n and the signal-to noise ratio ρ . For sufficiently small ψ ,

$$\nu_{opt} = \left\{ \frac{\left(1 - \frac{1}{M}\right)\psi^2}{3(1 + \rho)} \right\}^{\frac{1}{3}} \quad \mathcal{M}^{min} = \frac{3}{4} P_n (M + 1)(1 + \rho) \nu_{opt}$$

These results are new and important because they represent precise analysis of a non-stationary deterministic inverse modelling system problem. These results are in agreement with the form of the upper bounds for the misadjustment provided in [4] for the deterministic non-stationarity.

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(10)

SUPERIORITY OF LMS OVER RLS FOR TRACKING A CHIRPED SIGNAL

Odile Macchi¹ and Neil Bershad²

I. Introduction

When an adaptive filter, receiving coloured inputs, has to track a nonstationary environment, it is often said that the RLS algorithm will outperform the LMS one because it is known to converge faster. However, convergence speed is a transient property, independent of the amount of noise, while tracking is a steady-state performance and is therefore also influenced by the noise level. The answer is not obvious. There is in fact no single answer, and it depends very much on the problem under consideration. It has already been proved [1] that LMS can be superior in a context where the optimal filter $W_0(k)$ to be tracked is a zero-mean random function of time.

In this contribution we consider a case where $W_0(k)$ has deterministic time variations. A coherent signal $s(k)$ with power P_s is buried in additive white noise $n(k)$, with power P_n . It is known that the SNR can be improved by implementing a predictor based on past observed samples

$$X(k) = (x(k-1), \dots, x(k-M))^T; \quad x(k) = s(k) + n(k) \quad (1.1)$$

$$\hat{x}(k) = \hat{s}(k) = W^T X(k). \quad (1.2)$$

In fact, if we consider the errors

$$e(k) \triangleq x(k) - \hat{x}(k), \quad (1.3)$$

$$\eta(k) \triangleq s(k) - \hat{s}(k), \quad (1.4)$$

$$e(k) = \eta(k) + n(k), \quad (1.5)$$

the optimal estimator W_0 minimizing $E(|\eta|^2)$ is at the same time the optimal predictor minimizing $E(|e|^2)$, the noise sequence being independent.

When the sinusoid $s(k)$ is subject to chirping with a chirp rate ψ , according to

$$s(k) = \sqrt{2P_s} \exp i[k\theta + \frac{k^2}{2} \psi + \phi], \quad (1.6)$$

the optimal filter $W_0(k)$ is time-varying. It therefore has to be tracked, for example, by means of an adaptive algorithm of the kind

$$W(k) = W(k-1) + F(e(k), X(k) \dots), \quad (1.7)$$

$$e(k) \triangleq x(k) - W(k-1)^T X(k). \quad (1.8)$$

The system is depicted in Fig. 1. This specific example occurs in practical HF radars.

The filter input $x(k-1)$ is coloured since it contains a sinusoid. We are thus in the exact situation where it has always been assumed that RLS will outperform LMS. In what follows we show that this assumption is true in the case of a high input SNR, but in the situation of a poor SNR, it is wrong: LMS will outperform RLS unless the chirp rate ψ is very high. The limiting case corresponds to a fixed value c less than 1 of the so-called "non-stationarity degree", defined by

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COMPARISON OF RLS AND LMS ALGORITHMS FOR TRACKING A CHIRPED SIGNAL

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ABSTRACT

The Recursive Least Squares (RLS) algorithm is known to converge faster than the Least Mean Squares (LMS) algorithm when the environment is stationary and the input is colored. It is then often concluded that RLS will track better than LMS in a non-stationary environment. As an example of a non-stationary colored input, this paper studies a chirped sinusoid buried in additive white noise n . s can be adaptively recovered using an M -tap predictor W . When W is controlled by the RLS algorithm with forgetting rate $\beta = (1-\lambda)$, the output misadjustment \bar{m}_2 is dominated by a delay term of order β and a fluctuation term of order β . Hence, a value β_{opt} exists which yields a minimum value. Similar behavior by the LMS algorithm results in another minimum misadjustment. The ratio of the minima does not depend on the chirp rate ϕ but on M and the input SNR ρ . For a satisfactory predictor output SNR, LMS will track better than RLS (smaller misadjustment) unless 1) $\rho \gg 1$ or 2) ϕ is sufficiently large. Thus, this work leads to the very useful and unexpected result that LMS usually tracks better!

I. INTRODUCTION

Adaptive algorithms, which are capable of tracking a non-stationary input, always exhibit two contradictory features: (i) the convergence speed - a transient property which improves as the forgetting rate (say β) increases, (ii) the steady state fluctuations - due to measurement and algorithm noise which degrade the performance as β increases. It is often assumed that RLS tracks a nonstationary environment better than LMS because it has a greater convergence speed. However, this assumption is not sufficiently justified because ii) above is not considered. Indeed, tracking is not a transient problem but a steady-state problem. Thus, the measurement and algorithm noise are of primary importance. In this paper, a correct methodology is presented to deal with the tracking comparison of RLS and LMS for the non-trivial case of a noisy chirped sinusoid. The sinusoid is to be recovered by an adaptive predictor. For both algorithms, the misadjustment or residual mean square error of the adaptive filter output is properly minimized by optimizing the forgetting rate. The resulting β_{opt} , ν_{opt} depends on the noise and the chirping rate (ν is the step-size of LMS). The corresponding misadjustments M_{min}^{RLS} and M_{min}^{LMS} are then be compared as a function of ϕ , ρ , and M . In the most interesting case (small ρ , average ϕ) LMS is found superior.

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II. RECOVERING A NOISY CHIRPED SIGNAL

For a coherent signal $s(k)$ buried in white noise $n(k)$, the SNR can be improved by adding a number of samples M properly phase corrected by a transversal filter W . This is the basic idea behind the "adaptive line enhancer" shown in Fig. 1. The device is adaptive because the predictor weights are controlled by the (noisy) predictive error $e(k)$. The following notation is used:

$$\text{noisy signal: } x(k) = s(k) + n(k) \quad (2.1)$$

$$\text{past samples: } X(k) = [x(k-1), \dots, x(k-M)]^T \quad (2.2)$$

$$\text{predicted signal: } \hat{s}(k) = W^T X(k) \quad (2.3)$$

$$\text{recovery error: } n(k) = s(k) - \hat{s}(k) \quad (2.4)$$

$$\text{control error: } e(k) = x(k) - W^T X(k) \quad (2.5)$$

$$= n(k) + n(k) \quad (2.6)$$

The noise sequence $n(k)$, with power P_n , is assumed gaussian and independent. The signal is assumed to be a chirped sinusoid, namely

$$s(k) = \sqrt{P_s} \exp j[k\theta + \frac{k^2}{2}\phi + \phi] \quad (2.7)$$

where ϕ is a random phase with flat probability density, θ is related to the center frequency and ϕ , hereafter called "chirping rate", is responsible for the non-stationary character of the signal; P_s is the signal power and the input SNR is

$$\rho = P_s / P_n \quad (2.8)$$

Clearly Eq. (2.7) cannot hold for all k in $[-\infty, +\infty]$. However, it is a reasonable model inside a time interval T such that $|T\phi| \ll \pi$. Due to Eq. 2.6, minimization of the recovery error $E(|n|^2)$ is equivalent to minimization of the control error $E(|e|^2)$. The minimization can be performed recursively, using $e(k)$ to control the filter state W , using either the RLS (see Section III) or the LMS algorithm (see Section IV).

Consider the optimal filter $W_o(k)$ (time-varying when ϕ is nonzero) which should be achieved ideally by both algorithms. Using the notation

$$V = \text{diag}(e^{j\phi}, e^{j2\phi}, \dots, e^{jM\phi}) \quad (2.9)$$

$$D_o = [bc, b^2c^2, \dots, b^M c^M]^T \quad (2.10)$$

$$b \triangleq \exp j\phi; \quad c \triangleq \exp [-j\frac{\phi}{2}] \quad (2.11)$$

it has been shown in [3] that the optimal filter is given by

$$W_o(k) = R V^k D \quad (2.12)$$

$$K \triangleq \rho / (1 + M\rho) \quad (2.13)$$

EFFETS DE SATURATION SUR L'ALGORITHME LMS EN ANNULATION D'ECHO AVEC DONNEES BINAIRES

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La plupart des études sur l'adaptation LMS en annulation d'écho considèrent des données gaussiennes et/ou un algorithme opérant en mode linéaire. La réalité est quelque peu différente: - l'implantation numérique des algorithmes introduit des non linéarités, - les données sont souvent binaires. Dans cet article, un paramètre de saturation contrôle une non linéarité agissant sur l'erreur qui intervient dans l'algorithme LMS. Cette non linéarité peut ainsi varier de façon continue de la fonction signe (quantificateur à 1 bit) à la fonction linéaire (absence de quantification). Les résultats mesurés par l'écho résiduel montrent que, comparées à des données gaussiennes, des données binaires dégradent d'autant plus les performances que la saturation est élevée (proche de la fonction signe). La différence entre les nombres de bits de l'erreur nécessaires pour des données gaussiennes et binaires est évaluée pour un même écho résiduel en fonction de la puissance des données lointaines et du paramètre de saturation.

The effect of a saturation type error non-linearity in the weight update equation in LMS adaptive echo-cancellation is investigated for an independent binary data model. A nonlinear difference equation is derived for the mean norm of the difference between the estimate and the unknown filter to be estimated by the algorithm. The difference equation is evaluated numerically. It is shown that far-end binary data interference is much more deleterious to algorithm transient behavior than far-end gaussian data interference. The bit differential, for the same performance in a digital implementation of the algorithm, is studied for binary vs gaussian data as a function of the binary data power and the saturation parameter of the non-linearity.

I INTRODUCTION

Quelques articles récents [1-5] ont étudié l'algorithme LMS modifié par la présence de non linéarités dans l'incrément. Dans [2,3] des non linéarités de type saturation sont utilisées pour modéliser les effets dus à la précision finie dans des implémentations numériques de l'algorithme LMS. L'hypothèse de données gaussiennes faite lors de ces études n'est plus très réaliste dans le cas d'annulation d'écho de données.

Le cas de données binaires en annulation d'écho a été étudié dans [4,5] lorsque la fonction signe apparaît dans l'algorithme LMS. Il est montré que par rapport à des données gaussiennes, des données binaires de même puissance dégradent les performances de l'annulation d'écho (pour un même pas d'adaptation). Tel n'est pas le cas pour l'algorithme LMS classique où la densité de probabilité des données n'intervient pas.

Cet article étend les résultats de [2,4]. Il analyse les effets des données binaires, sur la vitesse de convergence de l'algorithme, lorsque le nombre de bits utilisés pour représenter l'erreur qui apparaît dans l'incrément de l'algorithme varie de 1 (fonction signe) à l'infini (cas linéaire). Les résultats permettent notamment d'évaluer le nombre de bits supplémentaires nécessaires lorsqu'on passe du cas gaussien au cas binaire pour une même performance d'annulation d'écho.

II ANALYSE

1. Algorithme LMS modifié

Le problème de l'annulation d'écho, illustré par la figure 1, représente un système de transmission bilatérale de données [4].

Le récepteur doit reconstituer les données lointaines $d_o(n)$ à partir d'une observation $d(n)$ qui est la somme de $d_o(n)$, d'un bruit additif $n_g(n)$ et d'un signal d'écho des données proches $x(n)$. Cet écho s'écrit $W_0^T X(n)$ où W_0 est un filtre inconnu de longueur N et $X(n)$ le vecteur des données proches

$$X^T(n) = (x(n), x(n-1), x(n-2), \dots, x(n-N+1)).$$

Pour supprimer l'écho on utilise un filtre adaptatif $W(n)$. Le signal d'erreur à l'entrée du récepteur peut s'écrire

$$e(n) = d_o(n) + n_g(n) - V^T(n) X(n) \quad (1)$$

où

$$V(n) = W(n) - W_0. \quad (2)$$

On suppose que les termes de l'équation (1) sont indépendants deux à deux. Les données proches $x(n)$ sont indépendantes, binaires, de puissance A et la longueur N du filtre adaptatif est suffisamment grande pour que la sortie $V^T(n)X(n)$ du filtre différence $V(n)$ soit considérée comme gaussienne [4].

Le signal d'erreur qui sert à piloter les coefficients de l'anneur est aussi le seul signal dont dispose le récepteur. Une restitution correcte de ne pourra donc être obtenue qu'après une bonne réduction de l'écho résiduel $V^T(n)X(n)$. Ceci peut être obtenu par l'algorithme LMS de pas d'adaptation μ :

$$W(n+1) = W(n) + \mu e(n) X(n). \quad (3)$$